Applegarth Primary School

Progression of Calculations

Introduction

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the "Development Matters" EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age and stage expectations

The calculation policy is organised according to age expectations as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at before being moved onto the next level as soon as they are ready. The essence of the mastery curriculum is that all children are working on the same objective (not that they all complete the same work). The emphasis should be on speedy catch up through effective intervention for those working below age related expectations so that they are working on the methods expected for their age by the end of the year.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

• Although the main focus of this policy is showing the core **Concrete, Pictorial and Abstract** ways of solving maths problems, it is important to recognise that the ability to calculate mentally lies at the heart of numeracy.

• Mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing.

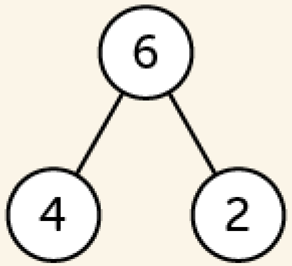
• Written recording both helps children to clarify their thinking and supports and extends the development of more *fluent* and sophisticated mental strategies.

• Children are encouraged to use the most efficient method for them, making sure they use ones they have a clear understanding of.

• The long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task. They should do this by always asking themselves:

- Can I do it in my head using a mental strategy?
- Could I use some jottings to help me?
- Should I use a written method to work it out?

Number Bonds

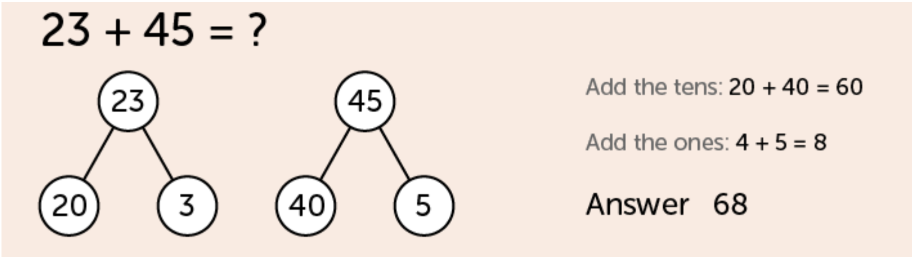


Number bonds refer to how numbers can be combined or split up, the 'part-part-whole' relationship of numbers.

When talking about number bonds in Singapore maths we are referring to how numbers join together and how they can be split up. A lot of emphasis is put into number bonds from the early year foundation stages so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

The mastery of number bonds is an important foundation required in subsequent mathematical learning and as a basis in the development of mental strategies. A strong number sense allows students to decide what action to take when trying to solve problems in their head.

An example of how a student would use number sense gained from number bonds to perform a mental calculation.



Good practice in primary mathematics: evidence from 20 successful schools November 2011

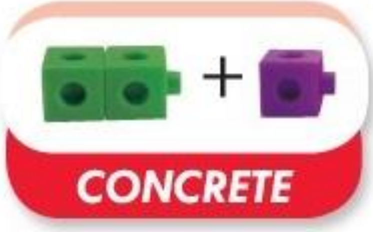
The following information has been taken from the Ofsted report ***Good practice in primary mathematics: evidence from 20 successful schools*** which can be downloaded here.

Concrete Pictorial Abstract (CPA) Approach

One of the key learning principles behind effective maths mastery is the concrete pictorial abstract approach, often referred to as the CPA approach.
The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations.

Concrete representation

The active stage - a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

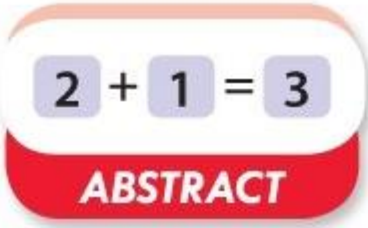


Pictorial representation

The iconic stage - a student has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

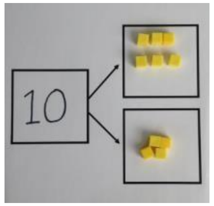
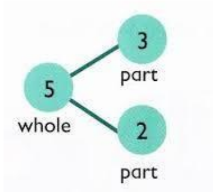
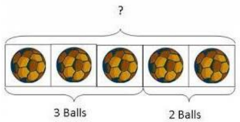
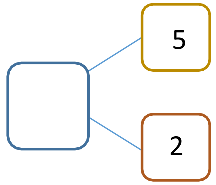
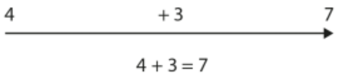
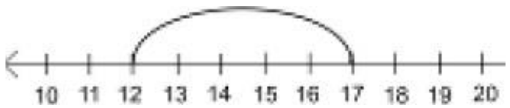
Abstract representation

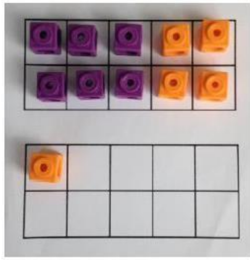
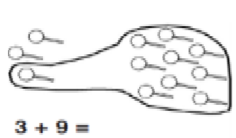
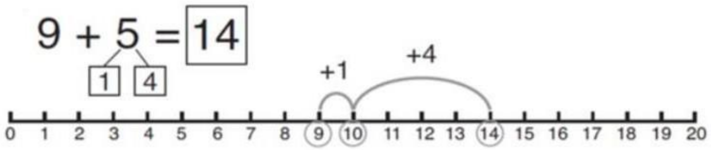
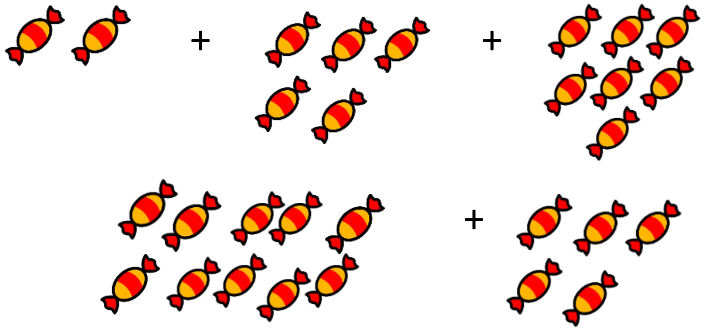
The symbolic stage - a student is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 this is the ultimate mode, for it is clearly the most mysterious of the three.

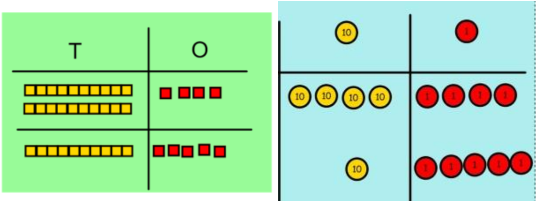
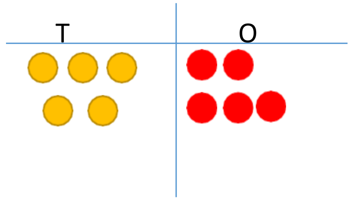
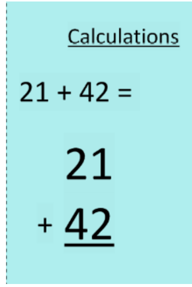
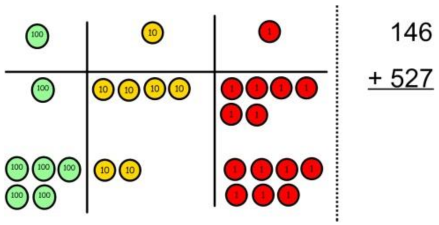
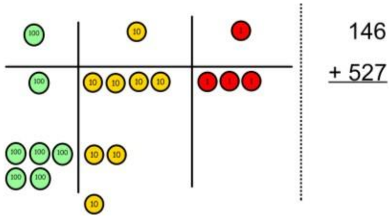
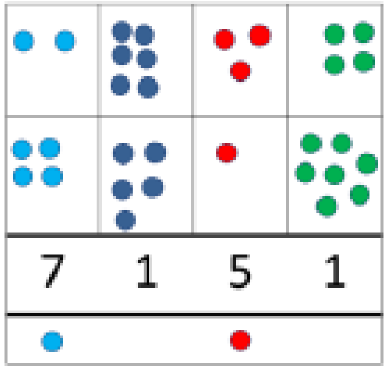
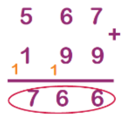
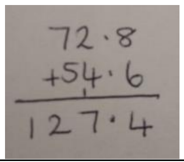
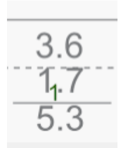


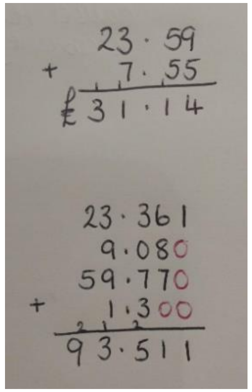
Progression in Calculations

Addition sum, total, parts and wholes, plus, add, altogether, more, regroup, rename, ‘is equal to’ ‘is the same as’

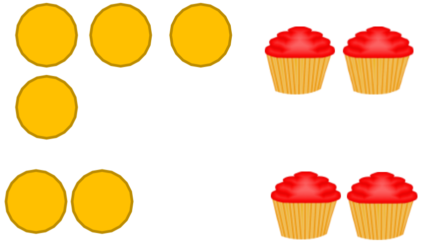
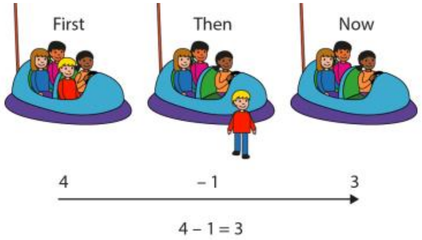
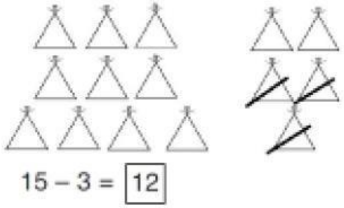
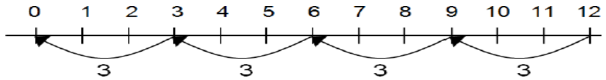
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model __________ is a whole, __________ is a part, __________ is a part. There are _______ in total. First… Then… Now…	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10 = 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on The bigger number is ________. To find the total, I need to start at the biggest number, then count **on**. (delete words as chn become more familiar)	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 First… Then… Now… E.g. **First** there were 4 children on the bus, **then** 3 children got on, **Now** there are 7 children on the bus.  12 + 5 = 17  Start at the bigger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer. ____ more than ____ is ____. The sum of ___ and ____ is_____. The total of ___ and ____ is ______.

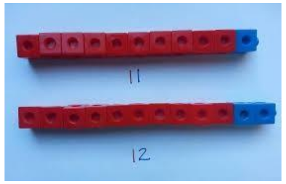
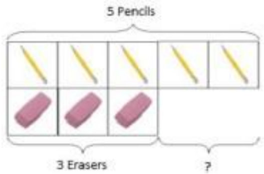
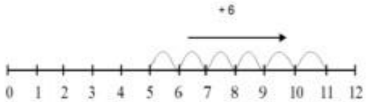
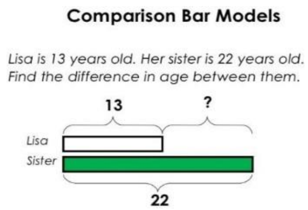
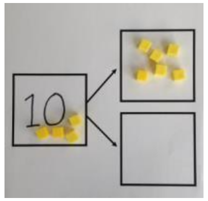
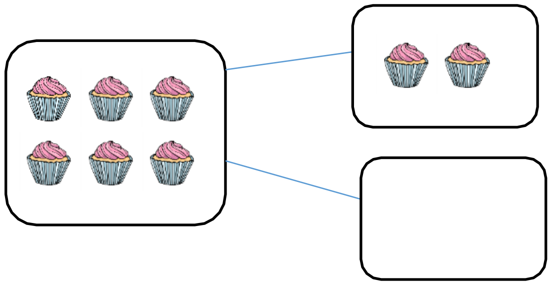
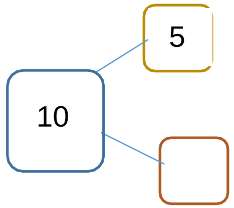
Regrouping to make 10. I need ____ to make ten. I have ____ left over. 10 + ___ is _____.	6 + 5 = 11   Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?
Adding three single digits ____ and ____ make ten. Ten add ____ is _____.	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.

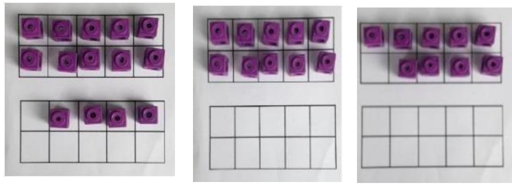
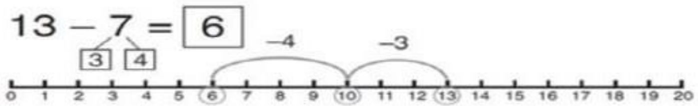
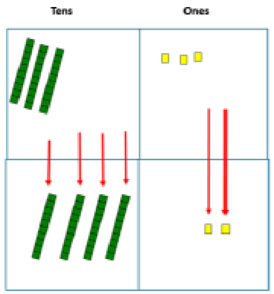
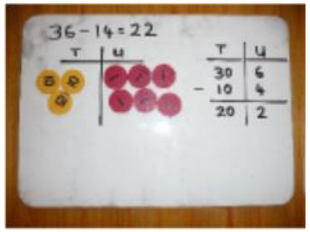
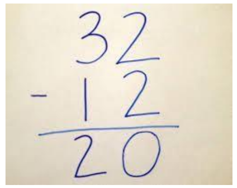
Column method- no renaming The___ is in the ones column, it represents ___ one(s). The ___ is in the tens column, it represents ____ ten(s).	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	
Column method- with renaming If the column sum is equal to ten or more, we must regroup. We need to regroup ten ones for one ten.	Make both numbers on a place value grid.  Add up the units and regroup 10 ones for one 10.  Add up the rest of the columns, regrouping the 10 counters from one column for the next place value column until every column has been added.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the renaming below the addition. $20 + 5$ $40 + 8$ $60 + 13 = 73$  As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. Year 5 and 6 include place holders. 

	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		

<u>Subtraction</u> take away, less than, the difference, subtract, minus, fewer, decrease, <u>regroup</u>, rename

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones First… Then… Now…	Use physical objects, counters, cubes etc to show how objects can be taken away. 6 – 2 = 4 4 - 2 = 2 	**First** there were 4 children in the car, **then** 1 child got out, **Now** there are 3 children in the car.  Cross out drawn objects to show what has been taken away. 	18 - 3= 15 8 – 2 = 6
Counting back The whole is ____. The part we are taking away is ______. Start on ______ and count back ____.	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4   Use counters and move them away from the group as you take them away counting backwards as you go.	Count back on a number line or number track  Start at the larger number and count back to the smaller number showing the jumps on the number line. This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Find the difference The difference is the amount between amounts.	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference.  Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Whole Model	 Link to addition - use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part whole model. 	Move to using numbers within the part whole model. 

Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Children should count below the number line	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without renaming The bigger number is _______ so that goes at the top. Take away the _______, then takeaway the _______.	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working.	$47-24=23$ $\begin{array}{r} 40+7 \\ -\ 20+4 \\ \hline 20+3 \\ \hline \end{array}$ This will lead to a clear written column subtraction. 

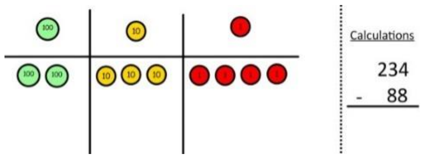
Column method with renaming

The ones column represents ___ one (s) minus ____ ones (s). This is equal to ___ ones. (repeat with tens, hundreds, etc)

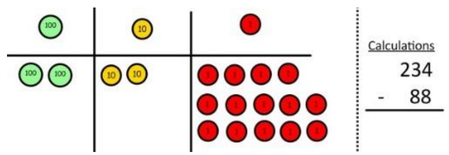
We cannot have two digits in any place value column, so we need to exchange one ten for ten ones.

Use Base 10 to start with before moving on to place value counters. Start with one regrouping before moving onto subtractions with 2 regrouping.

Make the larger number with the place value counters

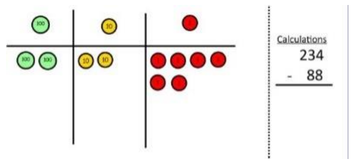


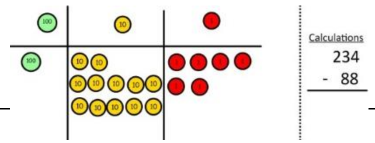
Start with the ones, can I take away 8 from 4 easily? I need to regroup one of my tens into ten ones.



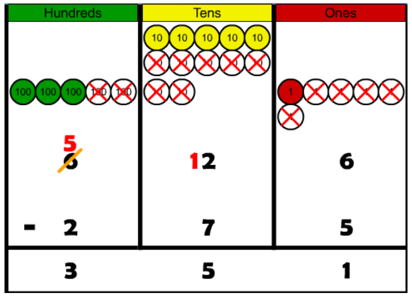
Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to regroup one hundred into ten tens.

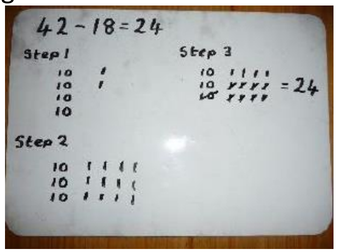




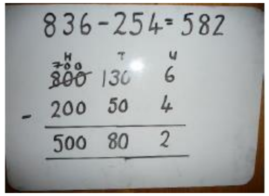
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the regrouping you do.



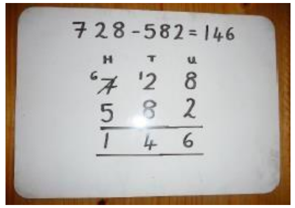
When confident, children can find their own way to record the regrouping.



Just writing the numbers as shown here shows that the child understands the method and knows when to regroup.



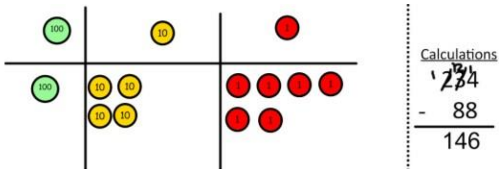
Children can start their formal written method by partitioning the number into clear place value columns.



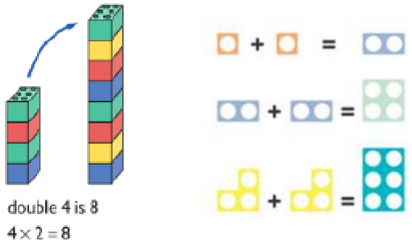
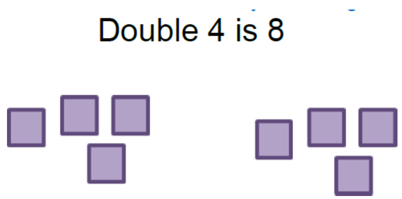
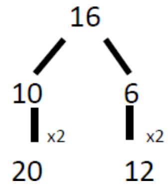
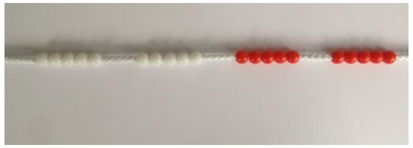
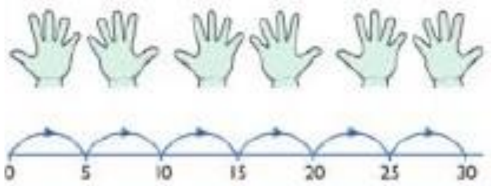
Moving forward the children use a more compact method.

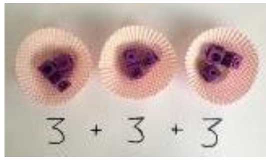
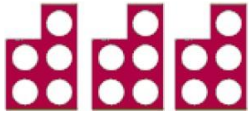
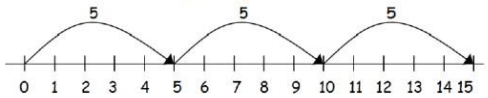
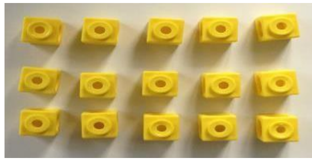
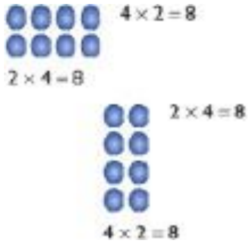
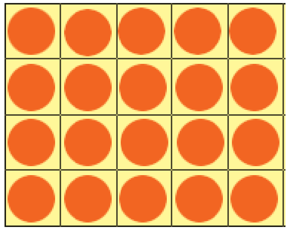
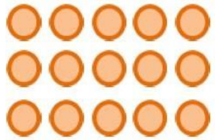
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{rcccc} & ^{5} & ^{12} & & ^{1} \\ & 2\ \cancel{6} & \cancel{3} & . & 0 \\ - & 2 & 6 & . & 5 \\ \hline & 2\ 3 & 6 & . & 5 \end{array}$$

	Now I can take away eight tens and complete my subtraction  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when regrouping and show where we write our new amount.		

Multiplication double, times, multiplied by, the product of, groups of, lots of, equal groups, regroup, rename

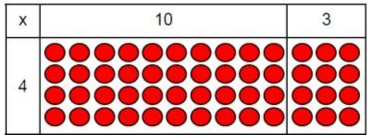
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling Doubling is an amount twice.	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.
Counting in multiples We are counting in multiples of ____ so we count every ____.	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition We are counting in multiples of ____ so we count every ____.	   Use different objects to add equal groups.	  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication ____ lots of ____ is the same as ____ lots of ____.	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Grid Method

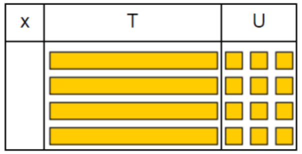
Because we are multiplying by ten, we need to add in a zero as a place value holder.

Show the link with arrays to first introduce the grid method.



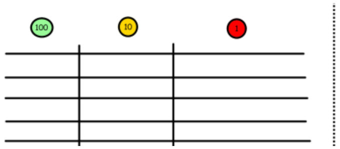
4 rows of 10
4 rows of 3.

Move on to using Base 10 to move towards a more compact method.



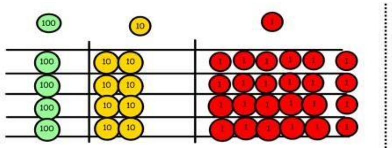
4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.
Fill each row with 126.

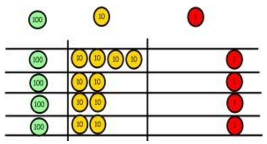


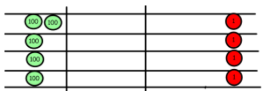
Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.



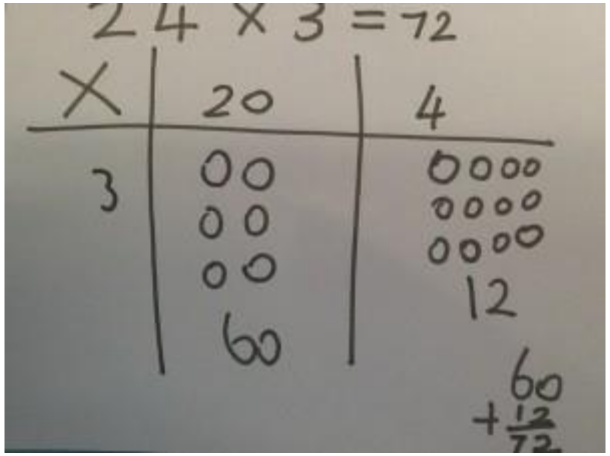
Calculations
4 x 126





Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

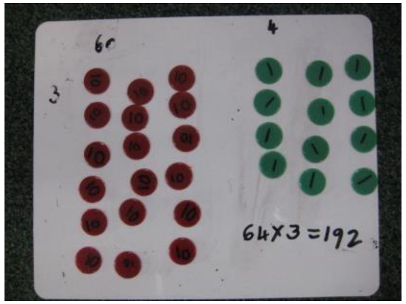
Column multiplication – with renaming

We always need to start at the ones.

____ ones times ____ ones is ___ ones.
____ ones times ____ tens is ____ tens.

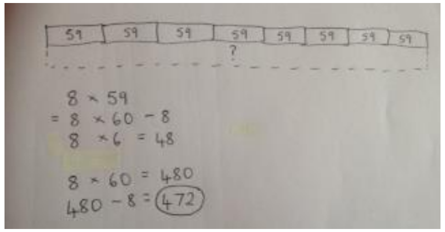
We cannot have more than one digit in any place value column, so we need to regroyp ___ ones as ____ ten

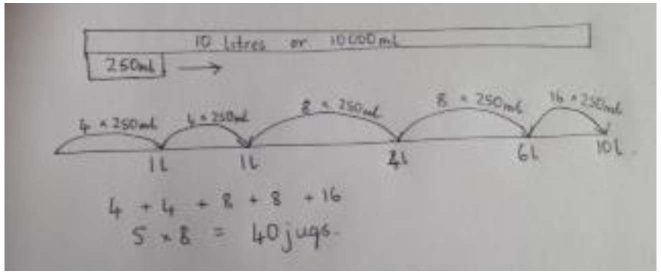
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



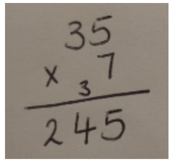


Recording formally

Moving forward split 35x7 as

30x7= 210
5x7= 35 +
245

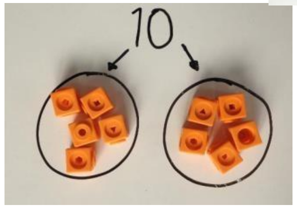
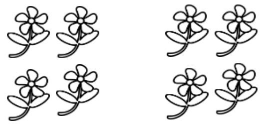
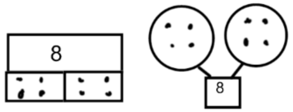
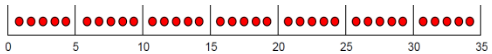
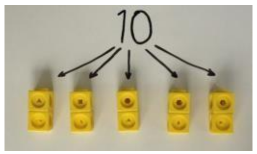
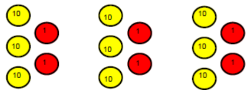
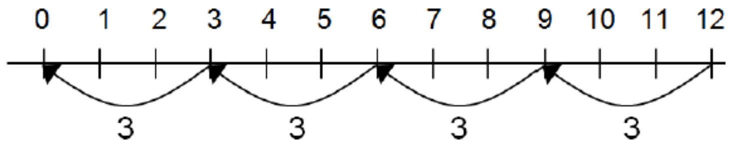
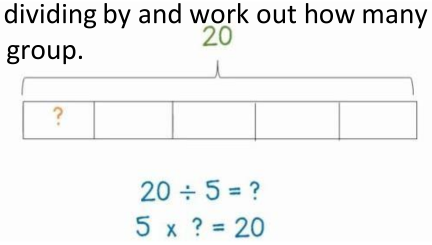
Finally

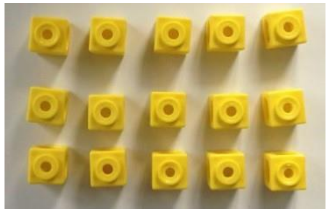


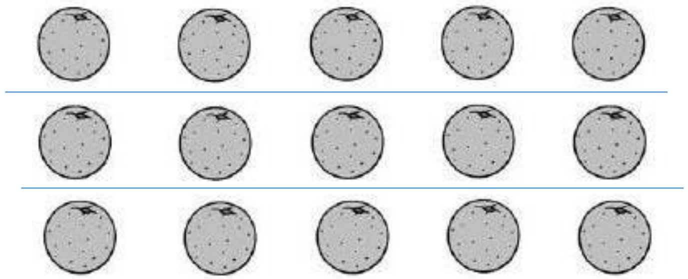
Regrouping figure on the line.

Long multiplication			Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 32 24x 128 (32x4) 640 (32x20) 768 This moves to the more compact method. 7 4 × 6 3 1 2 2 1 0 2 4 0 + 4 2 0 0 4 6 6 2 502 x 336 3012 15060 + 150600 168672

<u>Division</u> share, group, divide, divided by, half, divisor, dividend, quotient, remainder, <u>regroup</u>, rename

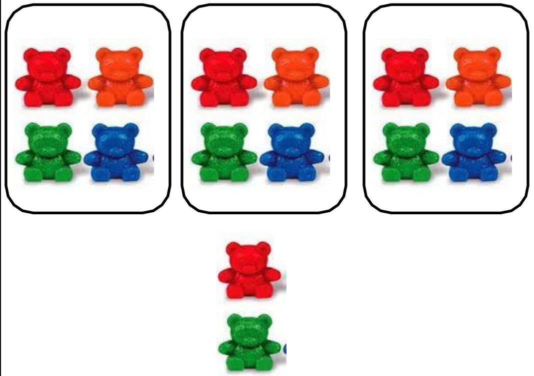
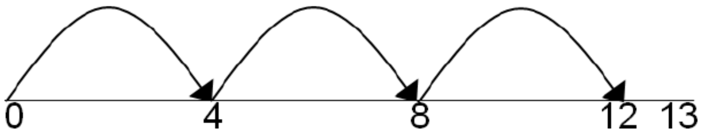
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups ____ shared equally between ____ is _____	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ 	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping ____ split into ___ groups means there would be ____ in each group.	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.    $96 \div 3 = 32$ 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. Repeated subtraction.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

<table>
<tr>
<td>Division within arrays</td>
<td>
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5   5 x 3 = 15
15 ÷ 5 = 3   3 x 5 = 15</td>
<td>
Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td>
</tr>
<tr>
<td>Division with a remainder

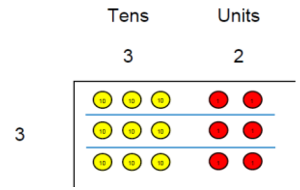
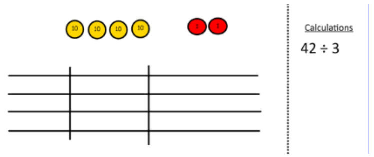
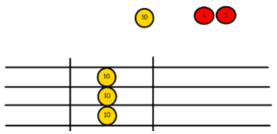
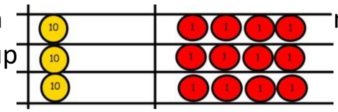
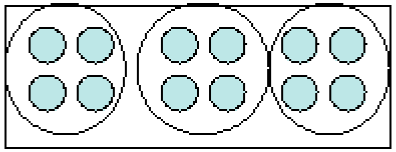
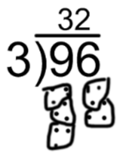
A remainder is what is left over after splitting into equal groups.

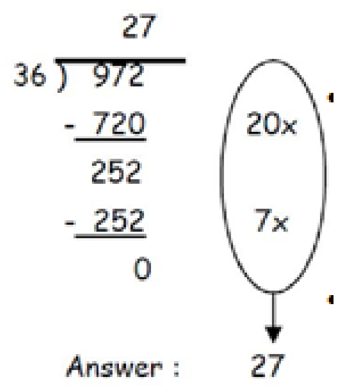
____ divided by ___ gives ____ equal groups, with _____ remaining.</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over
</td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Repeated addition. (The inverse could be done to support repeated subtraction.)


Draw dots and group them to divide an amount and clearly show a remainder.

</td>
<td>Complete written divisions and show the remainder using r.

</td>
</tr>
</table>

Short division	Use place value counters to divide using the bus stop method alongside:  42 ÷ 3=  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then mong the group  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. 	Begin with divisions that divide equally with no remainder. $\begin{array}{r\|l} & 2\quad 1\quad 8 \\ \hline 4 & 8\quad {}^{3}7\quad 2 \end{array}$ Move onto divisions with a remainder. $\begin{array}{r\|l} & 8\quad 6\quad r\quad 2 \\ \hline 5 & 4\quad {}^{3}3\quad 2 \end{array}$ Finally move into decimal places to divide the total accurately. $\begin{array}{r\|l} & 1\quad 4\quad .\quad 6 \\ \hline 3\quad 5 & 5\quad {}^{16}1\quad 1\quad .\quad {}^{21}0 \end{array}$

Long Division ThHTO / TO		Fact Box for 36 X2 = 72 X5 = 180 X10 = 360 X20 = 720 X40 = 1440 X100 = 3600 To be completed in the order as facts are needed Start with this for MOST pupils This idea can also be used for fining percentages of whole numbers.	Chunking  Fact box for 36 times tables needed using place value and doubling and halving knowledge. Can still do short division by using repeated addition to get multiples of divisor.